



**Inclusion in Europe through Knowledge and
Technology**

**Teaching Math to Students
who are Partially Sighted
Guidebook**

Inclusion in Europe through Knowledge and Technology

Information on the fundamental principles practices, educational material and teaching aids used to teach various subjects to students with special needs are few and far between. In some cases, material has been prepared for internal use at specialised schools or in other closed environments. In other cases, knowledge has been passed from teacher to teacher as part of workplace training.

No systematic material on pedagogical principles, practices, educational material and teaching aids exist for areas such as teaching first language teaching, foreign language teaching, mathematics and music for blind and partially sighted students.

With this in mind, the goal of this European project is to further develop, implement and disseminate good practices in the area of inclusive education and learning technologies by delivering three primary components: *Teaching Guides*, *Guide on good practices Inclusive learning and Teaching* and *SMART E-learning objects*.

Teaching Guides

In completing the project, RoboBraille partners have created a series of twelve educational guides covering fundamental principles, practices, educational material and teaching aids covering first language teaching, foreign language teaching, mathematics and music for the blind, partially sighted and dyslexic.

Inclusion Guide on good practices Inclusive learning and Teaching

In support of this, the project has collected and collated information on good inclusion practices in five select areas (teacher skills, alternate media, support structures, preparation for inclusion and teaching environments) which are published in a catalogue of good practices.

SMART E-learning

Finally, the project will adapt a comprehensive set of educational material on the RoboBraille service prepared in the LLL LdV RoboBraille SMART project into a set of learning objects for popular e-learning platforms for web and tablet deployment.

Introduction to this Teaching Guide

General principles

Since the challenges that visually impaired students have to face in learning mathematics are inherently the same in every school level, all of the methodologies and strategies presented in this guidebook can be applied with students in primary, secondary school and university. However, all the examples, educational scenarios and learning technologies taken into account in the following sections are related to primary school.

These guidelines are addressed in particular to:

- Teachers (mainstream and support, professional organisations of educators);
- School managers and supervisors;
- Healthcare professionals (social assistants, rehabilitators);
- School psychologists, Researchers of Inclusion;
- Institutional educational staff (legislators, policy makers and administrators);
- European disability organisations;
- Instructional designers;
- Designers of e-learning platforms.

Low vision/Partial sight

Low vision consists in a non-correctable vision loss that interferes with daily activities. It is defined in terms of function, rather than [numerical] test results. It takes into account both visual acuity and visual field.

Teachers must keep into account the variety of partially sighted students (different pathologies, central or peripheral visual difficulties, daily or nocturnal vision etc.) and therefore different strategies and recommendations must be selected for each of them.

Partially Sighted Students' teaching and learning approach is not the same as that for the blind students, however in both cases teachers know that total or partial lack of vision is not an obstacle to learning.

Unlike blind students, Partially Sighted Students have a partial command of the experiential world around them, often they can take advantage of the experience accumulated before the loss of sight.

Modern pedagogical criteria referred to students with disability are based on ICF International classification of functioning. ICF measures both health and disability, takes into account the context and therefore teachers must base their strategies on the functioning profile and not only on the disability of their students.

Teachers may evaluate the opportunity of programming an Individual Education Program (IEP) for each student with visual disability, based on the learner's abilities, needs, special requirements, prior learning, individual experiences, specific areas of strength and weakness. The IEP should include strategies based on each student's pace and learning style, being specifically goal-oriented. It needs a continuous re-adjustment according to the feedback received from the student (new skills, knowledge, attitudes) and his learning outcome.

Sensory-Perceptual Learning

Partially sighted persons spontaneously use alternative channels to acquire and re-organize information. Anticipatory function of sight: a full sighted person can "see" the door handle while he/she is approaching the door; the blind or partially sighted person must first go through the details of the door using a "touch mode" and only after decide what to do.

The perceptual process to acquire information goes through sequential steps, it is not a global, simultaneously visual approach as with "seeing" people.

Teachers should encourage this alternative mode and base teaching and learning strategies on the use of all the senses: visual (sight), auditory (hearing), kinaesthetic/tactile (touch and balance), gustatory (taste) and olfactory (smell). They represent an integrative support to help the student to acquire the missing details and re-adjust distorted information. Each student will have his own channel of preference and his own perceptual mode. The multisensory approach is recommended for ALL the students and is an inclusive strategy.

Narrative, descriptive mode. More complex situations, items, themes will be introduced during the lesson through a descriptive, narrative, mode to compensate for the lack of interaction with the real object, situation etc.

The support of life-like situations will help stimulate concept development and cognitive functions.

Specialised pedagogies associated with teaching Math to students who are partially sighted

Abstract concepts can be assimilated only through practical manipulative experiences.

From the early years of primary schools, students learn abstract concepts (e.g., quantity, sets, numbers, sequences, comparisons, etc.) through manipulative practical experiences with specific didactic material (e.g., abacus, slide rules) or real world objects (e.g., balls, boxes, etc.). Hence, practical experiences are crucial for developing mathematical competences.

Reading and writing maths notation is achieved through empowering visuo-spatial abilities.

Maths notation and operations are represented in a bi-dimensional layout. The spatial arrangements of notation elements (e.g., digits, symbols, operators, grids, etc.) suggest how to proceed (e.g., simplification between numerator and denominator, sum of two digits in the operations in column, the intersection set in the Euler-Venn diagrams, etc.)

Example

$$\begin{array}{r}
 376 \\
 + 289 \\
 \hline
 665 \\
 \hline
 11
 \end{array}$$

An arithmetic sum in a bi-dimensional layout

Working with geometric shapes is achieved through empowering visuo-spatial and visuo-constructive skills.

Learning to solve geometric problems that imply the ability to draw geometric figures and operate on them through abstract concepts (e.g., calculating the value of the perimeter of a polygon given the measure of the sides) requires visuo-spatial and visuo-constructive skills that can be empowered through an incremental didactic pathway. At each step of this pathway students learn how to draw points, lines, polygons, how to highlight the perimeter or the surface on a shape, and so on.

Challenges relating to the disability /specific learning difficulty

As illustrated in the previous section, in order to properly learn mathematics, students have to do many different activities: attending frontal lessons, doing practical experiences through manipulative activities, writing and reading bi-dimensional maths notation for taking notes or doing exercises, drawing geometric shapes, and more. These activities may pose specific problems for partially sighted students. The following subsections analyse the difficulties that partially students face in different working scenarios: attending the frontal lessons, doing practical experiences, in the assessment test and doing homework.

The Frontal lesson

The frontal lesson in the classroom is essential for learning because the teacher explains concepts and exercises according to a didactic pathway specifically designed for the class. The frontal lesson of mathematics is highly interactive. The teacher usually writes on a blackboard or on a digital interactive blackboard and explains aloud while writing. Moreover, students are often asked to answer questions based on mathematical expressions, shapes or diagrams that are drawn on the blackboard. Furthermore, students may be required to consult books or exercise books for tables or drawings that cannot be easily reproduced on the blackboard. These different activities that are often conducted at the same time may generate hindrances for partially sighted students.

Reading on the blackboard

According to the residual sight, some partially sighted students are able to read on the blackboard in large characters, whereas other students are not. However, even if a partially sighted student can read on the blackboard, maths notation is very difficult to be read.

Indeed, due to the bi-dimensional layout of maths notation, the student has to scan the blackboard by sight both horizontally and vertically. Students who have a limited field of view run into difficulty in reading expressions. One more problem concerns the continuous changing of the focus on the mathematical expressions. The teacher can focus a given element on the blackboard by pointing with the finger or with a pointer. For example, in an operation in column the teacher draws the attention on the units, then on the result, then on the decimals and so on. For a partially sighted student it is difficult to continuously find out the element being focused.

Example

$$\begin{array}{r} 376 \\ + 289 \\ \hline \end{array}$$

The digit 6 in the units column of the first number is enclosed in a red box.

$$\begin{array}{r} 376 \\ + 289 \\ \hline 5 \end{array}$$

The result 5 in the units column is enclosed in a red box.

$$\begin{array}{r} 376 \\ + 289 \\ \hline 5 \\ \hline 1 \end{array}$$

The result 1 in the tens column is enclosed in a red box.

Verbal description: Performing a columnar addition in three steps

Understanding mathematical expressions read ambiguously

The teacher usually reads aloud what is writing on the blackboard. This is of paramount help for partially sighted students who cannot read on the blackboard or who can read with difficulty. Nonetheless, it is common practice for teachers either to read aloud a mathematical expression partially or ambiguously. Moreover, many verbal explanations are dependent on the visual context. For example, the sentence "this one can be summed with this r" is ambiguous depending on the digits pointed by the finger of the teacher.

$$x^3$$

$$x_3$$

Reading "x three" is ambiguous. It may mean x to the power of three or x subscript three.

Taking notes

While the teacher is writing on the blackboard and explaining by speech, students take their notes. Visually impaired students run into difficulty especially in taking notes in mathematics because of different factors:

- the teacher often does not read everything is writing on the blackboard hence the student has to do a great effort for reading on the blackboard or asking a support teacher and then writing the notes. Such an effort diverts attention from concepts being explained to writing notes;
- the teacher does not write sequentially on the blackboard. For example, on the left side is stated the problem. The arithmetic operations for solving the problem are written on the right side. In the end, the solution is written on the bottom, under the problem statement. The visually impaired students write in large characters and cannot get a global glance at the text and operations arranged both on the blackboard and being written on paper while taking notes. Hence the student easily does mistakes or misses some pieces of information;

- a visually impaired student takes a longer time in sketching shapes than a sighted student. Hence, every time the teacher draws shapes, the visually impaired student will need a longer time to write correct notes.

Drawings

Visually impaired students may run into difficulty both in understanding and in sketching drawings. Understanding a drawing requires both the ability to see global properties and the ability to see details. If the field of view is limited by visual impairment, the global properties are not understood at the first glance, hence a long time is needed to explore the drawing many times and put together its parts. For what concerns understanding details such as, for example, the intersections among lines, text labels (e.g., the names of intersection points and the names of line segments) or the direction of small arrows, visually impaired students with limited visual acuity are not able to find these details unless they magnify the image.

Reading on the book

Maths books are rich with text explanations intertwined with mathematical expressions, tables, cognitive maps and drawings. Especially in primary school maths books, drawings play a very important role both in presenting concepts and in the exercises. Actually, drawings are often used in books as attention activators and for engaging young students in learning maths. As analysed above, visually impaired students run into difficulty in reading maths expressions and in understanding drawings at a glance. Even if the partially sighted student can magnify the printed characters on paper through lenses or portable magnifiers, reading and understanding a maths book turns out to be a time consuming, frustrating and highly demanding task for most visually impaired students.

Manipulative practical experiences

Learning abstract concepts in maths is a process based on practical manipulative experiences. These experiences are essential educational steps for all students regardless of sight impairment. Nonetheless, some of these experiences may pose problems to visually impaired students. Let us consider three examples that clarify three frequent kinds of problems: the inability to detect visual properties, fine-motor skills assisted by sight and inaccessible software.

the details of a drawing by magnifying and scanning horizontally and vertically require a longer time with respect to reading by getting a glance. Likewise, writing or drawing in large characters or assisted by a magnifier requires a longer time than by sight;

- how the exercises are presented to the students. The statement of maths exercises often contains mathematical expressions and drawings in addition to text. As discussed, drawings and mathematical expressions may pose many different problems;
- whether the exercises are computer-based. Exercises that have to be done by a computer (e.g., multiple choice questionnaires) need to be accessible through a magnifier or a screen reader.

Homework

Practicing maths after the lesson in the classroom is necessary to becoming skilled and confident in solving problems. In doing homework, visually impaired students face the problems related to writing maths expressions, reading exercises on the book and properly understanding drawings. In addition, whereas in the classroom the student can be supervised by a support teacher or by the class teacher, in doing homework the student is not always supervised by a sighted person. Hence, the student must be autonomous in reading the exercise, in writing maths and drawing shapes.

A description of suitable teaching methodologies practices

Adapting study material

Visually impaired students cannot easily read any kind of document either on paper or in digital format. Especially in mathematics, maths expressions and drawings pose further problems in understanding the content. Therefore, visually impaired students should be provided with study material in accessible and usable format. Due to the many differences in how a sight impaired person can rely on the residual sight for reading and writing, there is no general golden rule for adapting study material in accessible and usable formats. Hence, as a first step, the teacher should understand how the sight impaired student can rely on the residual sight for reading and writing. In particular, the teacher should understand:

- what character size fits the reading ability of the student. Usually, sight impaired students can read characters between a minimum and maximum character size;
- what character styles can be read;

- the favourite colours and contrast. There are many differences among visually impaired people. However, high contrast colours or white foreground over black background are often a good choice;
- the minimum and maximum distance at which the student can read characters or can explore the drawing of a shape;
- what are the best light conditions for facilitating reading by sight.

Once understood how the student can rely on her/his residual sight for reading and writing, the teacher can choose the appropriate adaptations for the study material. There are two main ways to provide sight impaired students with study material:

- large print on paper;
- in accessible and usable digital format.

Large print study material

Study material, including books, exercises and notes can be given to sight impaired students in large print on paper. In preparing these resources some guidelines should be followed:

- the size of the characters must be in the character size range that can be read through the residual sight of the student;
- the style of the characters must be legible. San-serif characters are usually a good choice;
- the interleave between lines must be sufficiently high in order to guarantee good legibility;
- the colours used in drawings and the contrast between colours must be the ones that the student can easily see;
- the mathematical expressions should be well separated from the surrounding text so that the visually impaired student can reduce reading mistakes.

Online resources on large print mathematics can be found at:

<http://www.teachingvisuallyimpaired.com>

<http://www.mathdrills.com>

Study material in digital format

In adapting a document containing maths expressions and drawings to be readable by sight impaired students, some basic guidelines should be taken into account:

- the document should be well-structured. For example, in MS Word documents, styles should be used for titles, the table object should be used to put tables in the document, bullet and ordered lists should be used to insert lists of items in the document;
- figures, math expressions and tables should be numbered. If these elements are numbered, it is easy to find them in the document and refer to a specific element while speaking;
- mathematical expressions should be inserted in the document as math objects. For example, in MS Word, a maths expression should be inserted in the document by choosing the Insert formula menu item. If maths expressions are mathematical objects, the sight impaired student can apply the favourite viewing style for maths. Instead, if maths expressions are inserted, for example, as images it is far more difficult to apply a style on all or parts of the expression and customize how math symbols are rendered;
- use scalable image formats to insert figures so that by zooming in our out the image quality does not change;
- add a caption to each image so that it is easier for the sight impaired student to understand its meaning;
- if a graphical element is generated through a source code or a value table, provide a link to the source file. For example, if a diagram is generated from an Excel spreadsheet, provide a link to the spreadsheet. For a partially sighted student, it may be easier to access the diagram through the source spreadsheet than on the resulting image of the diagram.

Accessible documents including maths expressions and images can be created by using MS Word and MathType by Design Science (<http://www.dessci.com>)

The frontal lesson

As discussed before, the frontal lesson is an essential educational experience for learning maths properly. The difficulties for visually impaired students can be overcome by teachers following the guidelines presented below.

Example b

Number	Square	Cube
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125

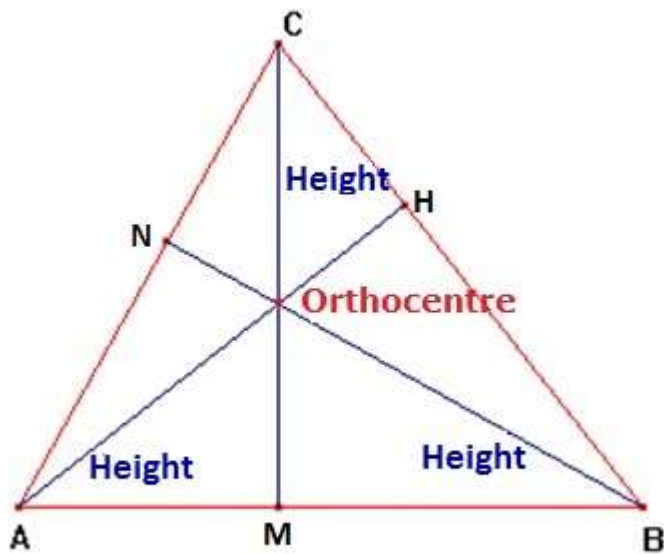
The table reports numbers from 1 to 5, their squares and cubes. In the first column on the left there are numbers from 1 to 5. In the second column there are squares: 1,4,9,16,25. In the third column there are cubes: 1,8,27,64,125

Describing drawings verbally

A verbal description of drawings on the blackboard is very important for facilitating the partially sighted student to understand the meaning of the drawing. The verbal description may be different from the one adequate for totally blind students. The teacher can assume that the student can see the basic features of the shape and that cannot see details (e.g. letters, intersection points of many different lines, etc.). Hence the teacher can give a brief description of the drawing and then focus on details. Some naming conventions should be used coherently in the description. For example, the vertices of a polygon are always read counter-clockwise; the vertex of an angle is always read in the middle (e.g. angle ABC has the vertex in B). A guide for describing drawings can be found at:

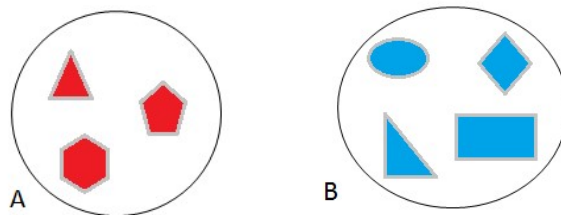
<http://www.artbeyondsight.org/handbook/acs-guidelines.shtml>

Example c



Verbal description: A triangle ABC with heights AH, BN, CM intersecting in Orthocentre

Example d



Verbal description: "The set A contains three elements, the set B contains 4 elements."

Providing notes

Since sight impaired students run into difficulty in taking notes, they can get advantage of notes provided by the teacher. It is more important that the visually impaired student pays attention to the explanation of the teacher and to the blackboard rather than trying to take notes meanwhile. Notes should be provided either in large print or in an accessible digital

format. If the notes are provided before the lesson (e.g. one day before the lesson) the student may read them in advance so as to be facilitated in understanding the frontal lesson. Moreover, notes can be consulted for doing homework autonomously. If the teacher cannot prepare the notes in advance before the lesson, the support teacher can prepare the notes during the lesson and give them to the student for enabling her/him to do homework autonomously.

Facilitating cooperation among students

Cooperative learning is fundamental for all students. It is even more useful for sight impaired students. Actually, on one hand a sight impaired student that works together with sighted students can ask a support for reading a mathematical expression or better understanding a complicated shape. On the other hand, the student can actively contribute in solving even difficult exercises. So, a fruitful and continuous exchange of competences is established among students. The teacher should promote cooperative learning by inviting students to do some exercises in pairs or in small groups. The teacher supervises the groups to monitor that each student actively participate and that especially the sight impaired student is actively involved in the working group.

Manipulative practical experiences

As analysed before, manipulative practical experiences are widely used for teaching mathematical concepts. In order to enable visually impaired students to take advantage of these experiences, teachers should design each task as a multisensory activity. The use of touch and hearing in addition to sight can compensate for low vision thus facilitating visually impaired students to understand the concepts conveyed through the manipulative activity. Also sighted students can benefit of multisensory activities because auditive and tactile feedback is a reinforcement for learning. It may be difficult for the teacher to transform specific visual experiences into multisensory practical experiences. In this particular case, the visual experience should be designed by carefully taking into account the residual sight of the sight impaired student. The manipulative practical experiences that are totally visual should be conducted in small working groups so that a sight impaired student can cooperate with sighted peers to do the activities. Manipulative practical experiences can be roughly divided into two groups: manipulative experiences with real world objects and with software applications. The following subsections examine these experiences by examples.

Example b

The teacher wants to introduce the operations on sets (e.g. union and intersection).

Two boxes containing objects with different shapes represent two sets. A third empty box is used to represent the union or the intersection set. The student puts in the third box the right objects belonging to union or intersection.

Note that are used different shapes instead of different colours so that they can be easily distinguished by sight impaired students, too.

Example c

The teacher wants to explain the amplitude of angles.

The class is divided in three-student groups. In each group one student represents the vertex of the angle and the other two students are placed along the sides of the angle. The sides of the angle are represented by a rope hold by the students. One of the students is placed on the side of the angle steps around the vertex and represents different angle amplitudes. Note that the visually impaired student should be the one who steps around the vertex in order to perceive the amplitude of the angle.

Learning technologies associated with the inclusive teaching of Math to Students who are partially sighted

Manipulative experiences with software applications

Nowadays there are many different educational programs for teaching maths and science that can be used to prepare and conduct manipulative experiences on digital objects (e.g., images, animations, digital sounds.). In order to design activities that are effective also for sight impaired students, the application adopted must be accessible through the assistive technology used by the student (e.g. magnifier, screen reader) and the activity must be designed taking into account the abilities in using digital tools and the residual sight of the sight impaired student. This guide contains references to accessible educational software. The following two examples illustrate two manipulative experience that can be conducted by using software applications.

Example a

The teacher wants to introduce geometric transformations (e.g. rotation and translation). A manipulative experience can be designed by using the software Geogebra (<http://www.geogebra.org>). It makes available many different tools for working with geometric shapes. Moreover, it is possible to adjust many different accessibility parameters, such as character size, style, colour, colour contrast, thickness of lines and dimensions of shapes. For example, the student can choose a shape and rotate or translate it both by using the mouse or by writing the number of degrees or units for the rotation or translation.

Example b

In recent years tablets have got widespread even among people with sight impairment thanks to their built-in accessibility features. On a touchscreen tablet it is possible to design a multimodal experience, even in maths education. There are many applications available for teaching maths on a tablet, but only few of them are accessible to sight impaired students.

One of these accessible applications is Math Melodies (<https://itunes.apple.com/it/app/math-melodies/id713705958?mt=8&Dettagli>) that makes available multimodal maths exercises, immersed in a tale, for primary school children. These exercises are accessible both to sighted, partially sighted and blind students. They cover the maths topics taught from the first to the fifth year of primary school. The teacher can use Math Melodies to let the students do manipulative experiences for example on counting exercises, sequences, arithmetic operations, and so on.

Assessment test

As discussed before the written assessment test may pose problems to visually impaired students. All of these problems can be overcome by adopting some strategies, hence the exercises and the evaluation objectives must not be changed for the sight impaired student.

Duration

The duration of the test should be calibrated on the abilities of the student. Hence, if the sight impaired student needs a longer time to achieve the test, due to the use of

compensatory tools, not to the lack of maths competences, it is good practice to let the student complete the test in a longer time, without reducing the number or changing the kind of the exercises.

The test sheet

The test sheet should be given to the sight impaired student in an accessible format agreed well in advance before the test (e.g., large print or in digital format). It is good practice to read aloud the test sheet before starting the test so that the student can check if something is not understandable. The student should be invited to express specific difficulties either in reading and especially in understanding the meaning of the drawings and maths expressions. If some difficulties are expressed, an additional verbal description of the drawings or of the maths expressions should be given to the student.

Computer-based tests

The accessibility of computer-based tests should be evaluated with the sight impaired student or accessibility specialists. Some test platforms are totally inaccessible to sight impaired students. If it is strictly indispensable to use these platforms, the test should be given to the sight impaired student in an alternative format and a sighted assistant should be in charge of completing the test on the platform according to the student's answers.

Homework

In order to enable the sight impaired student to be autonomous in doing homework, the teacher must ensure that:

- the student has the list of exercises to do at home. Hence, it is not sufficient to write the list of exercises on the blackboard, but it is necessary to read it aloud and give time to the student to take note. Alternatively, the list of homework exercises can be communicated to the class through a digital tool (e.g. by e-mail);
- the student can read the notes necessary for doing the homework. As discussed in previous sections, it is useful if some notes are given to the student in an accessible format by the teacher before or just after the lesson;
- the student can use at home all the assistive tools that are adopted in the classroom. For example, if the student uses a licensed software at school, it is necessary that she/he has the same license at home, as well.

